**Elementary Algebra**
**SAMPLE Final Examination**

NAME: _Key_

Fall 2014

You will have 2 hours to complete this exam. You may use a calculator but must show all algebraic work in the space provided to receive full credit. Read all directions carefully, simplify all answers fully, and clearly indicate your answer. Good Luck!

**Solve each equation. Show all algebraic work for full credit.** *(3 points each)*

1) $3 - x = 6$

$-3 \quad -3$

$\dfrac{-x}{-1} = \dfrac{3}{-1}$

$x = -3$

1) $\underline{X = -3}$

(section 2.1)

2) $-\dfrac{5}{2}x + 7 = 32$

$-7 \quad -7$

$-\dfrac{2}{5} \cdot -\dfrac{5}{2}x = 25 \cdot -\dfrac{2}{5}$

$x = \dfrac{-50}{5}$

$x = -10$

2) $\underline{X = -10}$

(Section 2.2)

3) $2 - 5y = 26 - y$

$+5y \quad +5y$

$2 = 26 + 4y$

$-26 \quad -26$

$\dfrac{-24}{4} = \dfrac{4y}{4}$

$-6 = y$

3) $\underline{y = -6}$

(section 2.2)

Distribute

4) $5(4 - 10y) = 30$

$20 - 50y = 30$
$-20 \qquad -20$

$\dfrac{-50y}{-50} = \dfrac{10}{-50}$

$y = -\dfrac{1}{5}$

4) $\underline{y = -\dfrac{1}{5}}$

(section 2.2)

5) $\dfrac{1}{3}x + \dfrac{2}{5} = \dfrac{3}{5}x$

$\dfrac{15}{1} \cdot \dfrac{1}{3}x + \dfrac{15}{1} \cdot \dfrac{2}{5} = \dfrac{15}{1} \cdot \dfrac{3}{5}x$

$\dfrac{15}{3}x + \dfrac{30}{5} = \dfrac{45}{5}x$

$5x + 6 = 9x$
$-5x \qquad -5x$

$\dfrac{6}{4} = \dfrac{4x}{4}$

$x = \dfrac{6}{4} = \dfrac{3}{2}$

5) $\underline{x = \dfrac{3}{2}}$

(section 2.2)

6) Distribute -1

$-6x - (-2x - 2) = -29 - 5x$

$-6x + 2x + 2 = -29 - 5x$

$-4x + 2 = -29 - 5x$
$+5x \qquad\qquad +5x$
_____
$x + 2 = -29$
$\quad -2 \quad -2$
_____
$x = -31$

6) $\underline{x = -31}$

(section 2.2)

7) $5(y - 7) = 3(y - 2) + 2y$

$5y - 35 = 3y - 6 + 2y$

$5y - 35 = 5y - 6$

$-5y \qquad -5y$

$-35 = -6$

Nonsense!

contradiction!
No Solution.

**Solve each inequality and graph the solution on the number line provided.** *(3 points each)*

8) $6x \geq 8x - 12$

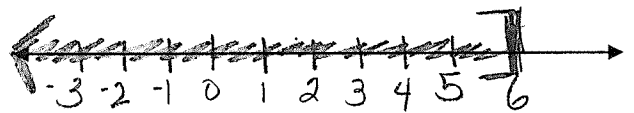$-8x \quad -8x$

$\dfrac{-2x}{-2} \geq \dfrac{-12}{-2}$

$x \leq 6$

\* reverse the direction of the inequality symbol when you multiply or divide by a negative.

8) $\underline{X \leq 6}$
(section 2.6)



9) $(3.5x + 8 - 2.41x) > (0.34x - 1.5 + 5)$  [·100 / ·100]

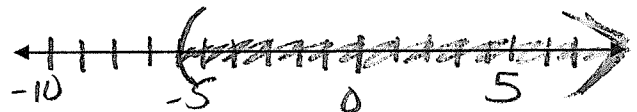\* Multiply every term by 100 to clear the decimals.

$350x + 800 - 241x > 34x - 150 + 500$

$109x + 800 > 34x + 350$

$-34x \qquad -34x$

$75x + 800 > 350$

$-800 \quad -800$

$\dfrac{75x}{75} > \dfrac{-450}{75}$
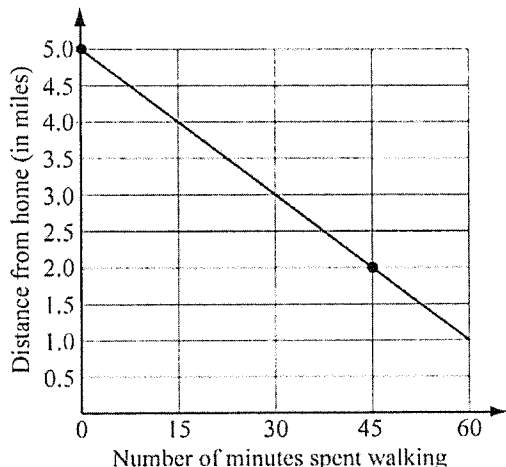
$x > -6$

9) $\underline{X > -6}$
(section 2.6)

10) Write the equation of a line that has a slope of $-\frac{1}{3}$ and intersects the y-axis at $(0, 7)$.  *(2 points)*

$$y = -\frac{1}{3}x + 7$$

(section 3.6)

11) Write the equation of the line for the graph shown.  *(2 points)*

$$m = \frac{-3}{45} \text{ or } \frac{-1}{15}$$

$$y = -\frac{1}{15}x + 5$$

Distance from home (in miles) vs. Number of minutes spent walking

(section 3.6)

12) Find the slope of each line.  Then state whether the two lines are parallel, perpendicular or neither. Show your work and justify your answer for full credit.  *(3 points)*

$y = \frac{1}{3}x + 2$

Slope of first line ___ $\frac{1}{3}$ ___

$3x + y = 4$
$-3x \quad -3x$
$y = -3x + 4$

Slope of second line ___ $-3$ ___

(Section 3.6)

Answer with reason: _The lines are perpendicular because the slopes are opposite signs and reciprocals._

13) Given the line $6x - 3y = 12$, find the following. *(1 point each)*

$$\begin{array}{c|c} X & y \\ \hline 2 & 0 \\ \hline 0 & -4 \end{array}$$

$\rightarrow 6x - 3(0) = 12$

$\dfrac{6x}{6} = \dfrac{12}{6}$

$x = 2$

$6(0) - 3y = 12$

$\dfrac{-3y}{-3} = \dfrac{12}{-3}$

$y = -4$

$\leftarrow$ OR $\rightarrow$

$\begin{array}{c|c} 6x - 3y = 12 & \\ -6x & -6x \\ \hline -3y = -6x + 12 & \\ \dfrac{-3y}{-3} = \dfrac{-6x}{-3} + \dfrac{12}{-3} \end{array}$
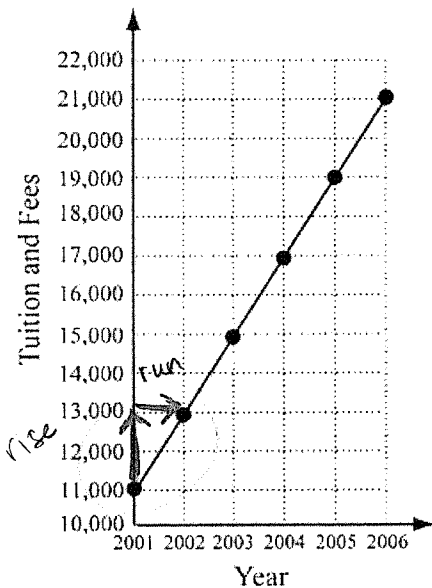
$y = 2x - 4$
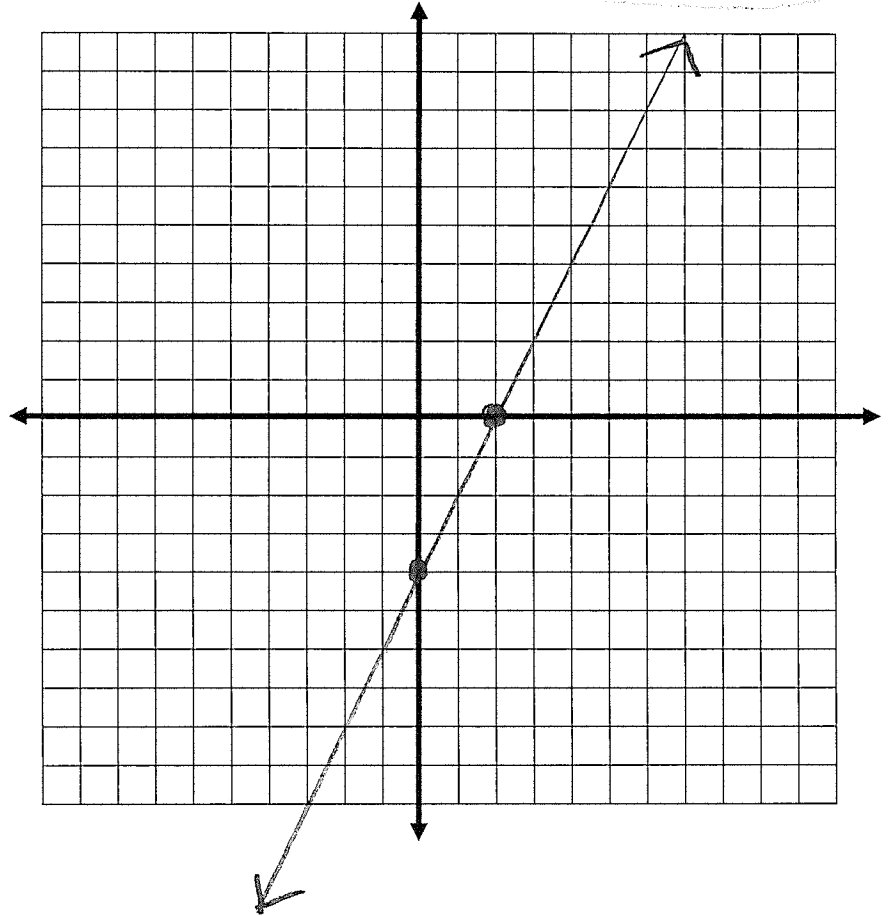
a) x-intercept: $(2, 0)$

b) y-intercept: $(0, -4)$

c) slope: $2$

d) Graph.

(section 3.3 and 3.6)

14) Use the graph to find the rate of change of the tuition and fees at private four-year colleges. *(1 point)*

$\rightarrow$ slope $= \dfrac{\text{rise}}{\text{run}} = \dfrac{\$2000}{1yr}$  (section 3.4)

Tuition and Fees vs Year

13) $\$2000/year$

**15) The value $v$ of a shopkeeper's inventory software program, in hundreds of dollars, is given by:**

$$v = -\frac{3}{4}t + 6$$

**where t is the number of years since the shopkeeper first bought the program.** *(1 point each)*

a)  Graph the equation.

b)  What was the initial value of the software program?  $\$600$

c)  Use the graph to estimate what the program is worth 4 years after it was first purchased. $\$300$

d)  Find the rate, in dollars per year, that the value of the software program was decreasing. $\$75/year$

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-300}{4 \text{ years}} = -\$75 \text{ per year}$$

initial value



4 years after it was purchased
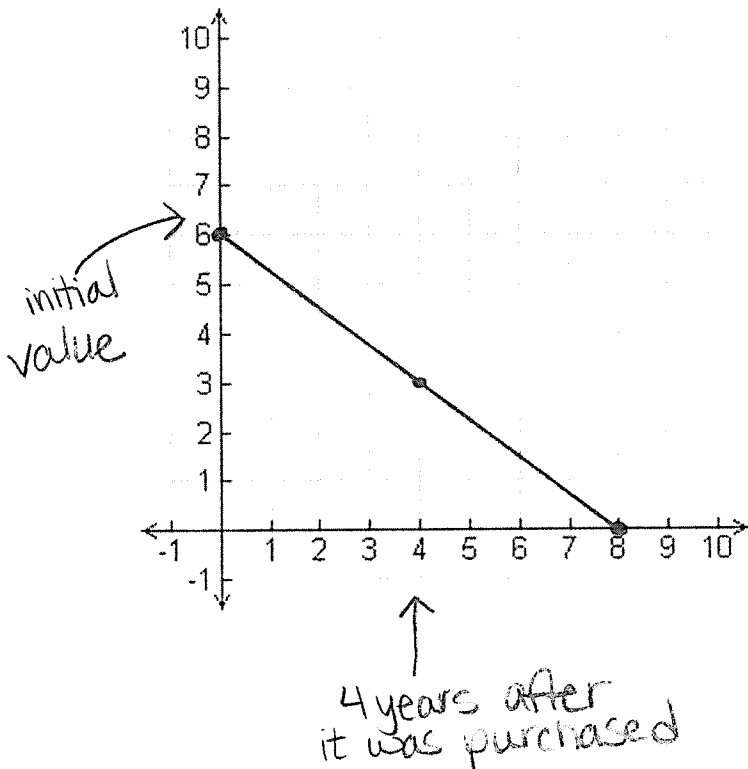
(section 3.2)

16) Write the equation of the line that contains the points (-4, 11) and (2, 5). *(3 points)*

$x_1 \ y_1 \qquad x_2 \ y_2$

$$m = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 11}{2 - (-4)} = \frac{-6}{6} = -1$$

$y = mx + b$ { Now use the slope and a point to find the y-intercept (b).

$5 = -1(2) + b \quad \leftarrow m = -1 \quad (2, 5)$
$\qquad\qquad\qquad\qquad\qquad\quad \overset{x}{\phantom{(}} \overset{y}{\phantom{)}}$

$5 = -2 + b$
$+2 \quad\quad +2$

$7 = b$

Equation! $y = -1x + 7$

(section 3.7)

17) If $f(x) = \frac{4x - 4}{2}$, then find $f(0)$ and $f(-1)$. *(1 point each)* (section 3.8)

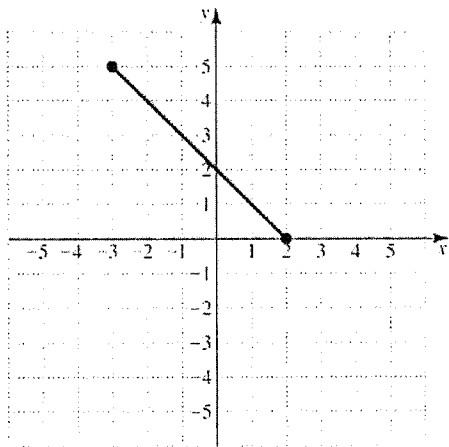(oops)

a) $f(0) = \dfrac{4(0) - 4}{2} = \dfrac{0 - 4}{2} = \dfrac{-4}{2} = -2$

17a) $\underline{\quad -2 \quad}$

b) $f(-1) = \dfrac{4(-1) - 4}{2} = \dfrac{-4 - 4}{2} = \dfrac{-8}{2} = -4$

17b) $\underline{\quad -4 \quad}$

18) What is the domain and range of the following function?   Write your answer in interval notation.
   *(1 point each)*   (Section 3.8)

Domain: $[-3, 2]$

(How far to the left and right does the graph go?)
(read this on the x-axis)

Range: $[0, 5]$

(What is the lowest point and highest point on the graph? Read this on the y-axis)

19) Find the point of intersection of the two lines using the <u>substitution</u> method.   (3 points)
   (Section 4.2)

$$3x - 2y = -17$$
$$y = x + 6$$

$$3x - 2(x + 6) = -17$$

$$3x - 2x - 12 = -17$$

$$x - 12 = -17$$
$$+12 \quad +12$$

$$x = -5 \rightarrow \text{plug this into either equation to find } y$$

$$(-5, 1)$$

$$y = -5 + 6$$

$$y = 1$$

20) Find the point of intersection of the two lines using the <u>elimination (addition)</u> method.  *(3 points)*

(section 4.3)

$-3(3x + y = 9)^{-3}$
$4x + 3y = 7$

$(4, -3)$

$-9x - 3y = -27$
$+ \ 4x + 3y = \ 7$

$\overline{\phantom{xxxxxxxxxxxx}}$

$\dfrac{-5x}{-5} = \dfrac{-20}{-5}$

$\boxed{x = 4}$

plug into either one
of the original equations

$3(4) + y = 9$
$12 + y = 9$
$-12 \qquad -12$

$\boxed{y = -3}$

21) Express 17,000,000,000 in scientific notation.  *(1 point)*

(section 5.2)

21) $\underline{1.7 \times 10^{10}}$

22) Write $6.13 \times 10^{-4}$ in standard form (decimal notation).  *(1 point)*

(section 5.2)

22) $\underline{0.000613}$

$.000613 = 0.000613$

4 places left

23) Multiply or Divide.  *(2 points each)*

a) $(1.3 \times 10^{18})(5 \times 10^{-12})$

$\boxed{6.5 \times 10^{6}}$

(section 5.2)

b) $\dfrac{9x^4 y^3}{12x^{-2} y^8}$

$\boxed{\dfrac{3x^6}{4y^5}}$

(section 5.1)

**Simplify each expression.  Write the result using positive exponents.  Please circle your final answer.**
*(2 point each)*  (Section 5.1)

24)  $x^8 \cdot x^{10} \cdot x^{-3}$

\* Same base...
  add the exponents.

$\boxed{x^{15}}$

25)  $(5^4)^8$

\* power to a power...
  multiply.

$\boxed{5^{32}}$

25)  $(-2x^5y^{-2})^3$

$(-2)^3 (x^5)^3 (y^{-2})^3$

$-8x^{15}y^{-6} = \boxed{\dfrac{-8x^{15}}{y^6}}$

**26)   Combine like terms: (2 points each)**

a)  $\boxed{9x^2} + 5x - 2 + 3x + 8 \boxed{- 5x^2}$

$\boxed{4x^2 + 8x + 6}$

(section 5.3)

b)  $(-6x^2 - 8x + 1) - (8x^2 - 4x + 7)$

$-6x^2 - 8x + 1 - 8x^2 + 4x - 7$

$\boxed{-14x^2 - 4x - 6}$

(section 5.4)

**Perform the indicated operations.  Simplify answers fully.  (2 points each)**

27)  $-4y(y^2 + 7y - 1)$      (section 5.5)

$\boxed{-4y^3 - 28y^2 + 4y}$

28) $(2x + 5)^2$ = $(2x+5)(2x+5)$ (section 5.6)

$$= 4x^2 + 10x + 10x + 25$$

$$= 4x^2 + 20x + 25$$

29) $(5p + 6)(5p - 6)$ (section 5.6)

$$25p^2 - 30p + 30p - 36$$

$$25p^2 - 36$$

30) $(4x - 3y)(5x - 2y)$ (section 5.7)

$$20x^2 - 8xy - 15xy + 6y^2$$

$$20x^2 - 23xy + 6y^2$$

31) $\dfrac{30x^4 - 24x^3 + 6x^2}{-6x^2}$ = $\dfrac{30x^4}{-6x^2} + \dfrac{-24x^3}{-6x^2} + \dfrac{6x^2}{-6x^2}$ (section 5.8)

$$= -5x^2 + 4x - 1$$

**Applications. Show your algebraic work for each problem. Include the proper units. Circle your final answer.**

32) A road rises 375.5 feet vertically over a horizontal distance of 7,221.2 feet. What is the grade of the road as a percent? Round to the nearest tenth of a percent. *(2 points)* (Section 3.1)

$$\text{Grade} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{375.5}{7,221.2} = .0519 = 5.19\%$$

$$\boxed{5.2\%}$$

33) A parking garage charges $5 to park plus $0.50 for each additional hour. Write the equation of the line that models this relationship. *(2 points)* (Section 2.5, 2.7, Chapters 3+4)

$$y = .5x + 5 \qquad \text{or} \qquad y = \tfrac{1}{2}x + 5$$

34) When all n teams in a baseball league play every other team twice, a total of G games are played, where $G = n^2 - n$. If a baseball league has 10 teams and all teams play each other twice, how many games are played? *(2 points)* (Section 2.3)

$$G = (10)^2 - 10$$
$$G = 100 - 10$$
$$G = 90 \text{ Games}$$

$$\boxed{90 \text{ games are played}}$$

35) You paid $53.40 for a meal including a 18% tip. How much was the cost before the tip? Round your answer to two decimal places. *(3 points)* (Section 2.5)

x = cost before the tip?

$$x + .18x = 53.40$$
$$\frac{1.18x}{1.18} = \frac{53.40}{1.18}$$
$$x = \$45.25$$

$$\boxed{\text{The cost before the tip was } \$45.25}$$

36) The equation $T = \frac{1}{4}N + 40$ can be used to determine the temperature T, in degrees Fahrenheit, given the number of times N that a cricket chirps per minutes. Determine the number of chirps per minute for a temperature of 84 °F. *(3 points)* (Section 2.5)

$$84 = \frac{1}{4}N + 40$$
$$-40 \qquad\qquad -40$$
$$(4)\,44 = \frac{1}{4}N \cdot \frac{4}{1}$$

$$176 = N$$

176 chirps per minute

37) At a school fundraiser, student tickets cost $2 each and adult tickets cost $5 each. A total of 396 tickets were sold and $1,167 was collected. How many student tickets were sold? How many adult tickets were sold? *(3 points)* (Section 4.4)

S = number of student tickets ($2)
a = number of adult tickets ($5)

$$-2(S + a = 396)-2 \qquad \begin{cases} -2S - 2a = -792 \\ +2S + 5a = 1167 \end{cases}$$
$$2S + 5a = 1,167$$

$$\frac{3a}{3} = \frac{375}{3}$$

a = 125 adult tickets
S = 271 student tickets

38) Karen's financial aid stipulates that her tuition not exceed $3,500. If her local community college charges an $80 registration fee plus $900 per course, what is the greatest number of courses for which Karen can register? *(3 points)* (Section 2.7)

X = number of courses

$$80 + 900x \le 3500$$
$$-80 \qquad\qquad -80$$
$$\frac{900x}{900} \le \frac{3420}{900}$$

$$x \le 3.8$$

The greatest number of classes she can register for is 3.