Intermediate Algebra

SAMPLE Final Exam for Fall 2017

NAME _Answer Key_

You will have 2 hours to complete this exam. You may use a calculator (TI-84 or lower, no cell phones) but must show all algebraic work in the space provided to receive full credit. Read all directions carefully, simplify all answers fully, and <u>clearly indicate your answer</u>. Good Luck!

Factor Completely. If the polynomial is prime, say so. (2pts each)

1. $3x^2 - 7x - 20$

$$(3x + 5)(x - 4)$$

2. $49 - 4x^2$

$$7^2 - (2x)^2 = (7 + 2x)(7 - 2x)$$

Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$ and, where appropriate, rationalize all denominators. (3 pts each)

3. $\dfrac{3x-12}{x+4} \div \dfrac{5x-20}{x^2-16} = \dfrac{3x-12}{x+4} \cdot \dfrac{x^2-16}{5x-20}$

$$= \dfrac{3(x-4)}{x+4} \cdot \dfrac{(x+4)(x-4)}{5(x-4)}$$

$$= \boxed{\dfrac{3(x-4)}{5}}$$

4. $\dfrac{x^2}{x-1} + \dfrac{1}{1-x}$

$$\dfrac{x^2}{x-1} - \dfrac{1}{x-1} = \dfrac{x^2-1}{x-1} = \dfrac{(x+1)(x-1)}{x-1} = \boxed{x+1}$$

$\boxed{10}$

Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$ and, where appropriate, rationalize all denominators. (3 pts each)

5. $\dfrac{1}{x-1} - \dfrac{1}{x+1}$, $\quad LCD = (x+1)(x-1)$

$$\frac{x+1 - 1(x-1)}{(x+1)(x-1)} = \frac{x+1-x+1}{(x+1)(x-1)} = \frac{2}{(x+1)(x-1)}$$

6. $\sqrt[5]{64x^{11}y^8} \;=\; \sqrt[5]{2^5(2)(x^2)^5 \cdot x \, y^5 \cdot y^3} \;=\; 2x^2 y \sqrt[5]{2xy^3}$

7. $\sqrt{\dfrac{7}{3}} \;=\; \dfrac{\sqrt7}{\sqrt3} \;=\; \dfrac{\sqrt7}{\sqrt3} \cdot \dfrac{\sqrt3}{\sqrt3} \;=\; \dfrac{\sqrt{21}}{3}$

8. $5\sqrt{32} - 7\sqrt{50} \;=\; 5\sqrt{4^2(2)} - 7\sqrt{5^2(2)}$

$$= 5(4)\sqrt2 - 7(5)\sqrt2$$

$$= 20\sqrt2 - 35\sqrt2 = -15\sqrt2$$

2

(12

Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$ and, where appropriate, rationalize all denominators. (3 pts each)

9. $(3 - \sqrt{2})(5 + \sqrt{2})$

$$15 + 3\sqrt{2} - 5\sqrt{2} - 2 = 13 - 2\sqrt{2}$$

10. $(7 - 8\sqrt{7}) - (12 - 11\sqrt{7})$

$$7 - 8\sqrt{7} - 12 + 11\sqrt{7} = -5 + 3\sqrt{7}$$

11. $(2 + 3i)(4 - 7i)$

$$8 - 14i + 12i - 21i^2$$
$$8 - 2i - 21(-1)$$
$$8 - 2i + 21 = 29 - 2i$$

12. $\dfrac{5}{4 - 3i}$ $= \dfrac{5(4+3i)}{(4-3i)(4+3i)} = \dfrac{5(4+3i)}{(4)^2 - (3i)^2} = \dfrac{5(4+3i)}{16 - 9i^2}$

$$= \dfrac{5(4+3i)}{16+9} = \dfrac{5(4+3i)}{25} = \dfrac{4+3i}{5} = \dfrac{4}{5} + \dfrac{3}{5}i$$

3

(12

State the domain of each function below. (1pt each)

13. $f(x) = 2x^2 - 7x + 5$

$$\text{Domain} = \text{All real numbers} = (-\infty, \infty) = \{x \mid x \in \mathbb{R}\}$$

14. $f(x) = \dfrac{x-3}{x^2-6x+5} = \dfrac{x-3}{(x-5)(x-1)}$; $x \neq 5$ and $x \neq 1$

$$\text{Domain} = \text{All real numbers except 5 and 1} = \{x \mid x \neq 5 \text{ and } x \neq 1\}$$
$$= (-\infty, 1) \cup (1, 5) \cup (5, \infty)$$
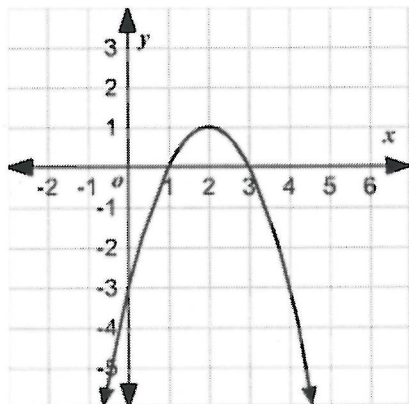
15. $f(x) = \sqrt{x-3}$

$$x - 3 \geq 0$$
$$x \geq 3$$

$\text{Domain} = \{x \mid x \geq 3\} = $ All real numbers that are greater or equal than to 3. $= [3, \infty)$

16. (2pts) Given $f(x) = 3x^2 - 7x + 9$, find $f(-2)$

$$f(-2) = 3(-2)^2 - 7(-2) + 9$$
$$= 3(4) + 14 + 9 =$$
$$= 12 + 14 + 9$$
$$\boxed{f(-2) = 35}$$

4

17. Use the graph of the function below to determine the following.

a). Does the function have a maximum or a minimum value? And what is that value? (2pts) __Maximum Value = 1__

b) What is the domain of the function? (1 pt) __All real numbers $= (-\infty, \infty)$__

c). What is the range of the function? (1 pt) __$(-\infty, 1]$__

d). What are the zeros of the function? (2 pts) __1 and 3__

18. For the quadratic function $f(x) = x^2 + 6x + 5$, find the following and graph. (2pts each)

Vertex (h, k): $h = \dfrac{-6}{2(1)} = -3$; $k = (-3)^2 + 6(-3) + 5$

$= 9 - 18 + 5 = -4$
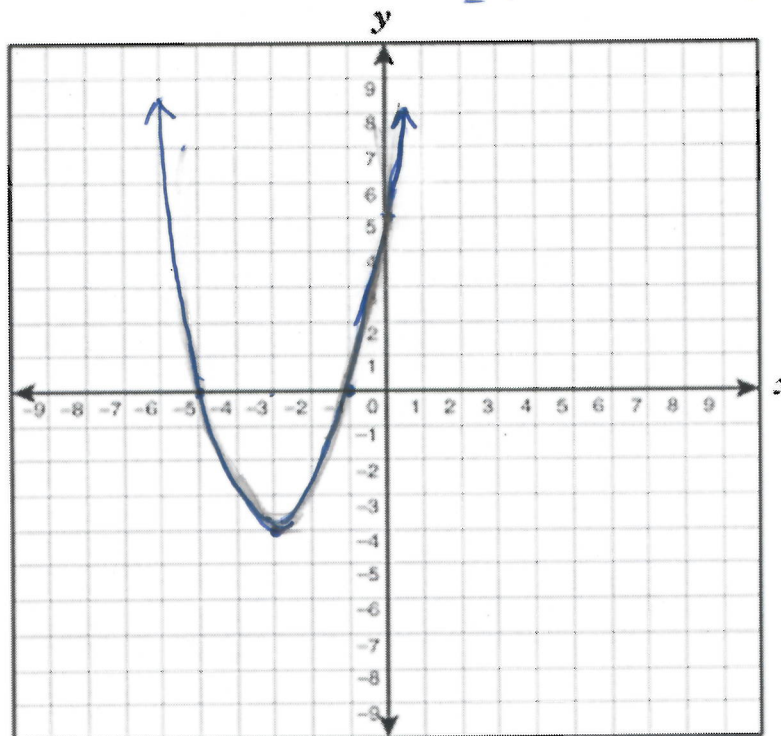
a). Vertex __$(-3, -4)$__

b). x-intercept(s) __-5 and -1__

$(x+5)(x+1) = 0$

$x + 5 = 0$ $x + 1 = 0$

$x = -5$ $x = -1$

c). y-intercept __5 or $(0, 5)$__

d). Graph to the right

5

14

Match the graph to the type of function that best describes it. The same type may be used multiple times or not at all. (2pts each)
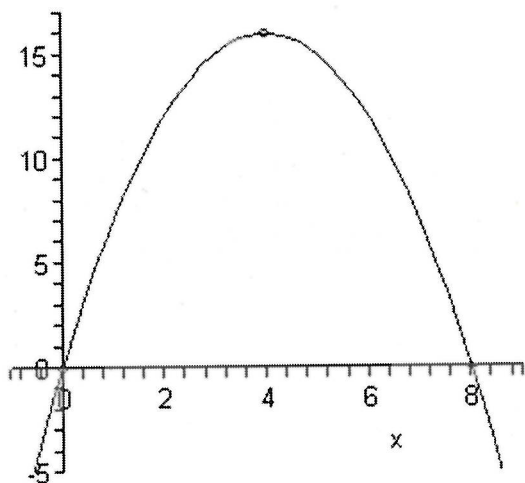
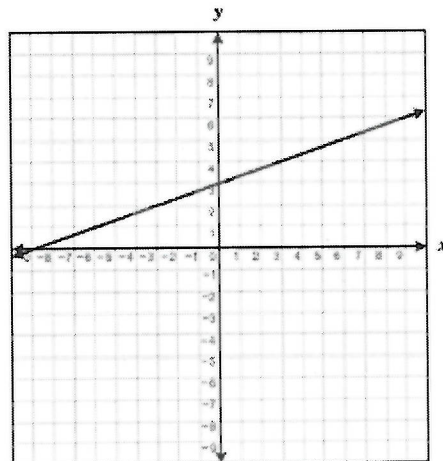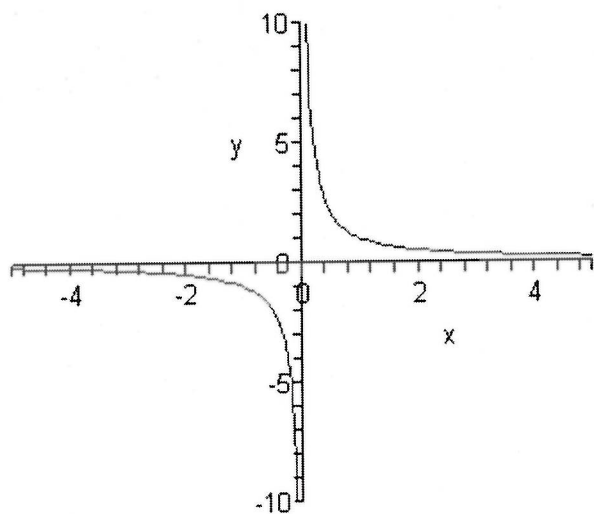a). Linear b). Quadratic c). Exponential d). Radical e). Rational
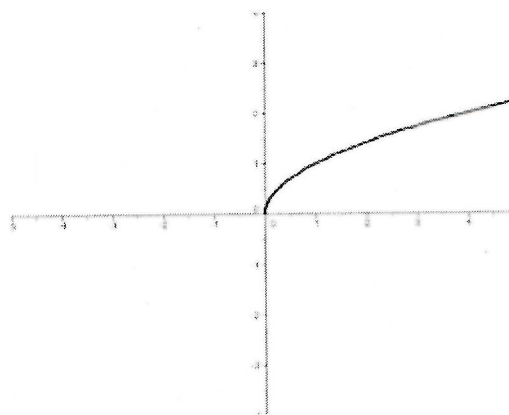
19. _Quadratic (b)_

20. _Linear (a)_

21. _Rational (e)_

22. _Radical (d)_

Solve each equation below. Simplify completely, do not round. (3 pts each)

23. $8x^2 - 3 = 10x$

$$8x^2 - 10x - 3 = 0$$

$$(4x+1)(2x-3) = 0$$

$4x+1 = 0$ or $2x - 3 = 0$

$4x = -1$ $2x = 3$

$\boxed{x = -\frac{1}{4}}$ $\boxed{x = \frac{3}{2}}$

24. $x = 6 + \sqrt{x}$

$$x - 6 = \sqrt{x}$$

$$(x-6)^2 = (\sqrt{x})^2$$

$$x^2 - 12x + 36 = x$$

$$x^2 - 13x + 36 = 0$$

$$(x-9)(x-4) = 0$$

$x - 9 = 0$ or $x - 4 = 0$

$\boxed{x = 9}$ or $x = 4$ (extraneous)

$\uparrow$ only solution

25. $\dfrac{-5}{x+2} + \dfrac{3}{x-2} = \dfrac{2x}{x^2-4}$

$$\frac{-5}{x+2} + \frac{3}{x-2} = \frac{2x}{(x+2)(x-2)} \; ;$$

Restriction

$x \neq \pm 2$

$$-5(x-2) + 3(x+2) = 2x$$

$$-5x + 10 + 3x + 6 = 2x$$

$$-2x + 16 = 2x$$

$$16 = 4x$$

$$\boxed{4 = x}$$

7

9

Solve each equation below. Simplify completely, do not round. (3 pts each)

26. $\sqrt[7]{x - 3} + 2 = 1$

$$\sqrt[7]{x - 3} = -1$$

$$\left(\sqrt[7]{x-3}\right)^7 = (-1)^7$$

$$x - 3 = -1$$

$$\boxed{x = 2}$$

27. $x^2 - 4x + 53 = 0$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(53)}}{2(1)} = \frac{4 \pm \sqrt{16 - 212}}{2}$$

$$= \frac{4 \pm \sqrt{-196}}{2} = \frac{4 \pm i\sqrt{196}}{2} = \frac{4 \pm 14i}{2} = \boxed{2 \pm 7i}$$

8

/6

Application Problems. For all problems where an equation is not given, you need to define your variable(s), set up an algebraic equation or equations, solve algebraically, and answer the question with the proper units. If an equation is given, be sure to answer the question completely and with proper units. **(4 pts each)**

28. Suppose H(x) = $-15x^2 + 75x + 10$ gives the height H in feet of a soccer ball x seconds after it is kicked. How long has the soccer ball been airborne if a fan sitting 100 feet above ground catches it on its way back down?

$$-15x^2 + 75x + 10 = 100$$
$$-15x^2 + 75x - 90 = 0$$
$$-15(x^2 - 5x + 6) = 0$$
$$-15(x - 3)(x - 2) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x - 2 = 0$$
$$\boxed{x = 3} \checkmark \qquad\qquad x = 2$$

$$\boxed{x = 3 \text{ Seconds}}$$

29. Suppose p(x) = $-x^2 + 980x - 3000$ gives the total profit in dollars made by a car dealership for selling x cars.

a). How many cars must be sold to maximize profit? (2 pts)

$$x = -\frac{b}{2a} = \frac{-980}{-2} = 490$$

$$\boxed{x = 490 \text{ Cars}}$$

b). What is the maximum profit? (2 pts)

$$p(490) = -(490)^2 + 980(490) - 3000 = \text{Max profit}$$
$$= -240,100 + 480,200 - 3000$$

$$\boxed{\text{Max profit} = p(490) = \$237,100}$$

9

8

30. The height of a rectangular wall is 7 feet shorter than the horizontal distance that it spans. If the wall measures 13 feet diagonally, then find the height and length of the wall.

$$x^2 + (x+7)^2 = (13)^2$$
$$x^2 + x^2 + 14x + 49 = 169$$
$$2x^2 + 14x + 49 - 169 = 0$$
$$2x^2 + 14x - 120 = 0$$
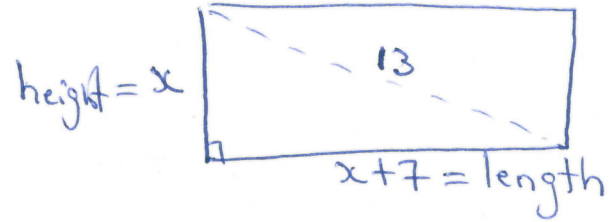$$2(x^2 + 7x - 60) = 0$$
$$2(x+12)(x-5) = 0$$
$$x + 12 = 0 \longrightarrow x = -12$$
$$x - 5 = 0 \longrightarrow \boxed{x = 5}$$

height = x [rectangle with diagonal 13, h, x+7 = length]

Length of wall is: __12 feet__

Height of wall is: __5 feet__

31. Kyle bikes 15 mph slower than Ellen does. Kyle travels 50 miles in the same time that Ellen travels 80 miles. Find the speed of each person.

$x = $ Ellen's speed ; $x - 15 = $ Kyle's speed ; $t = $ time

Ellen's time $= t = \dfrac{80}{x}$; Kyle's time $= t = \dfrac{50}{x-15}$

$$\frac{80}{x} = \frac{50}{x-15}$$

$$80(x-15) = 50x$$
$$80x - 1200 = 50x$$
$$-1200 = -30x$$
$$x = 40$$

Time $= \dfrac{\text{Distance}}{\text{Speed}}$

Ellen's speed: __40 mph__

Kyle's speed: __25 mph__

32. Chelsea can rake her lawn twice as fast as Eric. If they work together, it takes them 5 hours to rake the lawn. How long would it take Eric to rake the lawn alone?

$x = $ Chelsea's time ; $2x = $ Eric's time in hours

$$\frac{1}{x} + \frac{1}{2x} = \frac{1}{5} \quad ; \quad LCD = 10x$$

$$10x \left(\frac{1}{x} + \frac{1}{2x} \right) = 10x \left(\frac{1}{5} \right)$$

$$10 + 5 = 2x$$

$$15 = 2x$$

$$x = \frac{15}{2} \text{ hours}$$

Chelsea: $\frac{15}{2}$ hrs or 7.5 hrs

Eric: 15 hours

33. The height of a triangular coffee field is 1 mile shorter than its base. If the surface area of the field is 3 square miles, then find the base and height of the coffee field.

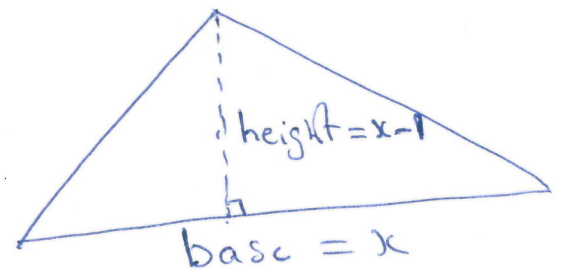$$(base)(height) = 2(Area)$$

$$x(x-1) = 2(3)$$

$$x^2 - x = 6$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x - 3 = 0 \quad or \quad x + 2 = 0$$

$$x = 3 \qquad\qquad x \neq -2$$

height $= x-1$

base $= x$

Base of coffee field is: ___3 miles___

Height of coffee field is: ___2 miles___

Bonus (3pts)

Simplify $\sqrt[3]{12 - \sqrt{18 - \sqrt[5]{30 + \sqrt[4]{16}}}}$ completely to its exact value.

$$= \sqrt[3]{12 - \sqrt{18 - \sqrt[5]{32}}}$$

$$= \sqrt[3]{12 - \sqrt{18 - 2}}$$

$$= \sqrt[3]{12 - 4} \qquad = \sqrt[3]{8} = \boxed{2}$$

Have a Wonderful Summer Recess!