Intermediate Algebra NAME Answer Key

~~**Final Exam Fall 2017**~~

SAMPLE Final Exam Spring 2018

You will have 2 hours to complete this exam. You may use a calculator (TI-84 or lower, no cell phones) but must show all algebraic work in the space provided to receive full credit. Read all directions carefully, simplify all answers fully, and clearly indicate your answer. Good Luck!

Factor Completely. If the polynomial is prime, say so. (2pts each)

1. $2x^2 - 7x - 15$

$(2x+3)(x-5)$

2. $4 - 9x^2$

$(2+3x)(2-3x)$

Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$ and, where appropriate, rationalize all denominators. (3 pts each)

3. $\dfrac{2x-10}{x+5} \div \dfrac{8x-40}{x^2-25}$

$$\frac{2(x-5)}{x+5} \cdot \frac{(x+5)(x-5)}{8(x-5)} = \frac{2(x-5)}{8} = \frac{x-5}{4}$$

4. $\dfrac{x^2}{x-7} + \dfrac{49}{7-x}$ $= \dfrac{x^2}{x-7} - \dfrac{49}{x-7} = \dfrac{x^2-49}{x-7}$

$$= \frac{(x+7)(x-7)}{x-7} = x+7$$

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Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$ and, where appropriate, rationalize all denominators. (3 pts each)

5. $\frac{3}{x-5} - \frac{2}{x+2}$

$$\frac{3(x+2)-2(x-5)}{(x-5)(x+2)} = \frac{3x+6-2x+10}{(x-5)(x+2)} = \frac{x+16}{(x-5)(x+2)}$$

6. $\frac{5}{2x} - \frac{4}{3x}$

$$\frac{15}{6x} - \frac{8}{6x} = \frac{7}{6x}$$

7. $\frac{40}{\sqrt{5}} = \frac{40\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}} = \frac{40\sqrt{5}}{5} = 8\sqrt{5}$

8. $2\sqrt[3]{16} - 4\sqrt[3]{54}$

$$2\sqrt[3]{2^3(2)} - 4\sqrt[3]{3^3(2)}$$

$$2(2)\sqrt[3]{2} - 4(3)\sqrt[3]{2}$$

$$4\sqrt[3]{2} - 12\sqrt[3]{2} = -8\sqrt[3]{2}$$

$\frac{}{12}$

Perform the indicated operation and simplify completely. Leave complex answers in the form $a + bi$ and, where appropriate, rationalize all denominators. (3 pts each)

9. $(4 + \sqrt{3})(5 - \sqrt{3})$

$20 - 4\sqrt{3} + 5\sqrt{3} - 3 = 17 + \sqrt{3}$

10. $\frac{x-2}{2} - \frac{x+1}{3}$

$\frac{3(x-2) - 2(x+1)}{6} = \frac{3x - 6 - 2x - 2}{6} = \frac{x-8}{6}$

11. $(3 - 2i)(5 - 4i)$

$15 - 12i - 10i + 8i^2$

$15 - 22i - 8 = 7 - 22i$

12. $\frac{3}{4+5i} = \frac{3(4-5i)}{(4+5i)(4-5i)} = \frac{12-15i}{(4)^2 + (5)^2} = \frac{12-15i}{16+25}$

$= \frac{12-15i}{41} = \frac{12}{41} - \frac{15}{41}i$

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State the domain of each function below. (1pt each)

13. $f(x) = -3x^2 + 2x + 1$

Domain = All real numbers $= (-\infty, \infty) = \{x \mid x \in \mathbb{R}\}$

14. $f(x) = \frac{x+2}{x+9}$

$x+9 \neq 0$; $x \neq -9$

Domain = All real numbers except -9 $= \{x \mid x \neq -9\}$

$= (-\infty, -9) \cup (-9, \infty)$

15. $f(x) = \sqrt{x+5}$

$x+5 \geq 0$; $x \geq -5$

Domain $= \{x \mid x \geq -5\}$ = All real numbers greater than or equal to -5.

16. **(2pts)** Given $f(x) = 3x^2 - 4x + 5$, find $f(-1)$

$f(-1) = 3(-1)^2 - 4(-1) + 5$

$= 3(1) + 4 + 5 = 12$

$f(-1) = 12$

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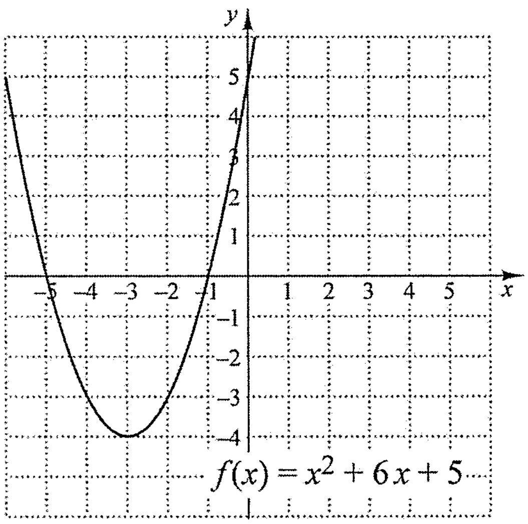
17. Use the graph of the function below to determine the following.



a). Does the function have a maximum or a minimum value? And what is that value? **(2pts)** Minimum Value = −4

b) What is the domain of the function? **(1 pt)** Set of all real numbers.

c). What is the range of the function? **(1 pt)** $[-4, \infty)$

d). What are the zeros of the function? **(2 pts)** −5 and −1

18. For the quadratic function $f(x) = x^2 + 2x - 3$, find the following and graph. **(2pts each)**

Vertex (h, k) $h = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$; $k = f(-1) = (-1)^2 + 2(-1) - 3 = -4$

a). Vertex $(-1, -4)$

b). x-intercept(s) −3 & 1

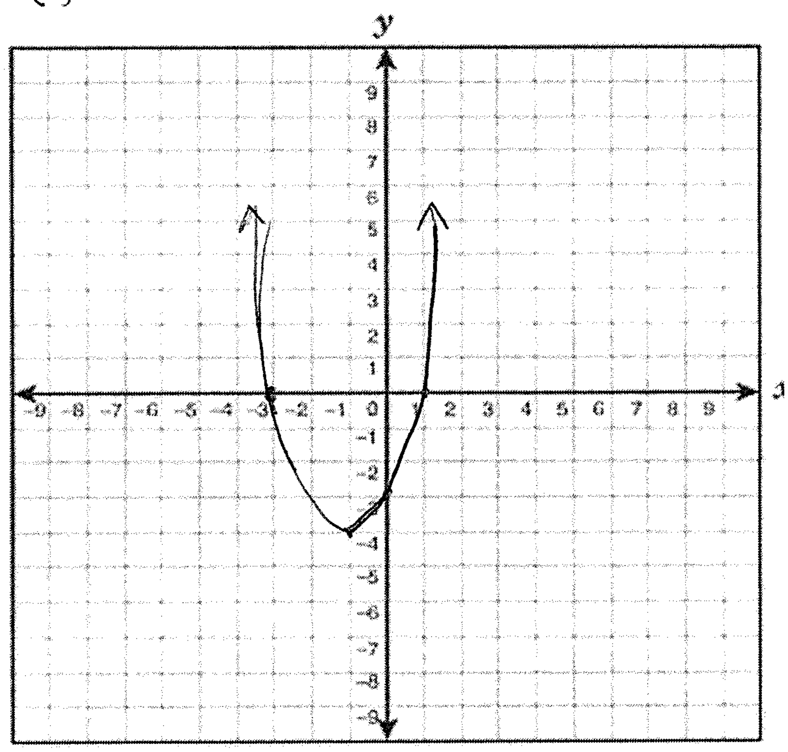
$(-3, 0)$ & $(1, 0)$

$(x+3)(x-1) = 0$

$x = -3$; $x = 1$

c). y-intercept −3 or $(0, -3)$

d). Graph to the right



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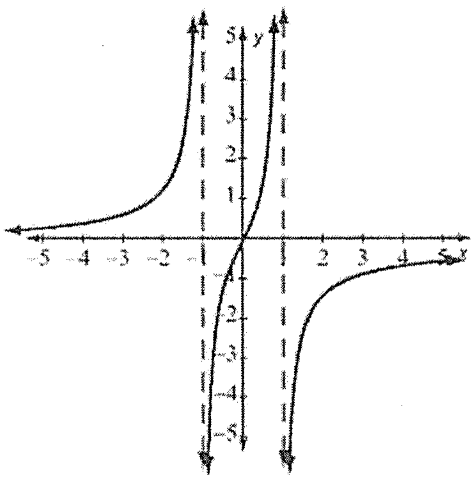
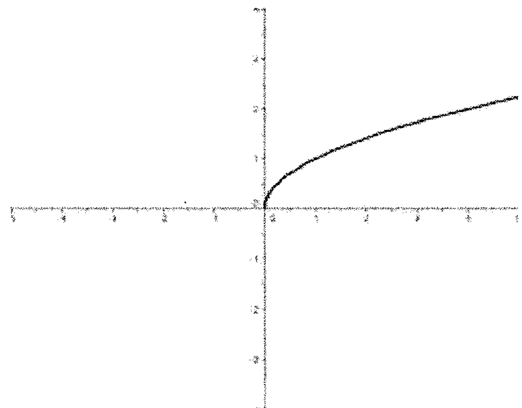
Match the graph to the type of function that best describes it. The same type may be used multiple times or not at all. (2pts each)

a). Linear b). Quadratic c). Exponential d). Radical e). Rational

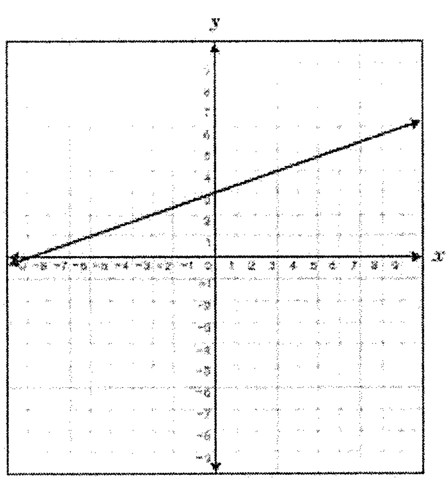
19. Rational



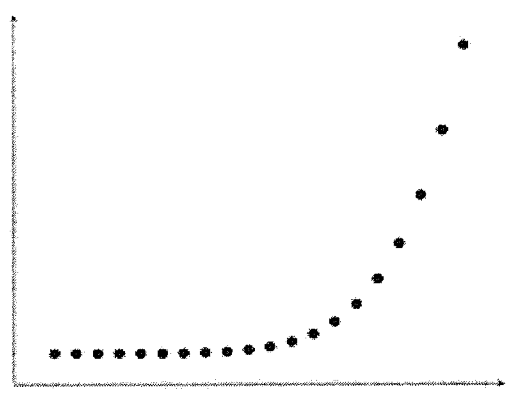
20. Radical



21. Linear



22. Exponential



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Solve each equation below. Simplify completely, do not round. (3 pts each)

23. $2x^2 - 5x = 12$

$2x^2 - 5x - 12 = 0$

$(2x+3)(x-4) = 0$

$2x + 3 = 0$ or $x - 4 = 0$

$2x = -3$

$x = 4$

$x = -\frac{3}{2}$

24. $x = 7 + \sqrt{x-5}$

$x - 7 = \sqrt{x-5}$

$(x-7)^2 = (\sqrt{x-5})^2$

$x^2 - 14x + 49 = x - 5$

$x^2 - 15x + 54 = 0$

$(x-6)(x-9) = 0$

$x - 6 = 0$ or $x - 9 = 0$

$x = 6$ or $x = 9$

Extraneous

Only solution.

25. $\frac{-4}{x-7} = \frac{2x}{x+3}$; Restriction: $x \neq 7$; $x \neq -3$

$2x(x-7) = -4(x+3)$

$2x^2 - 14x = -4x - 12$

$2x^2 - 14x + 4x + 12 = 0$

$2x^2 - 10x + 12 = 0$

$2(x^2 - 5x + 6) = 0$

$2(x-3)(x-2) = 0$

$x - 3 = 0$ or $x - 2 = 0$

$x = 3$ or $x = 2$

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Solve each equation below. Simplify completely, do not round. (3 pts each)

26. $x - \frac{14}{x} = 5$; Restriction: $x \neq 0$

$x^2 - 14 = 5x$

$x^2 - 5x - 14 = 0$

$(x-7)(x+2) = 0$

$x - 7 = 0$ or $x + 2 = 0$

$x = 7$ or $x = -2$

27. $x^2 - 4x + 8 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = 1$; $b = -4$; $c = 8$

$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(8)}}{2(1)} = \frac{4 \pm \sqrt{16 - 32}}{2} = \frac{4 \pm \sqrt{-16}}{2}$

$x = \frac{4 \pm 4i}{2} = 2 \pm 2i$

$x = 2 \pm 2i$

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Application Problems. For all problems where an equation is not given, you need to define your variable(s), set up an algebraic equation or equations, solve algebraically, and answer the question with the proper units. If an equation is given, be sure to answer the question completely and with proper units. (4 pts each)

28. Suppose H(x) = $-x^2 + 5x + 14$ gives the height H in feet of an arrow x seconds after it is launched. How long has the arrow been airborne if it is intercepted 20 feet above ground on its way up?

$H(x) = 20 \iff -x^2 + 5x + 14 = 20$

$-x^2 + 5x - 6 = 0$

$-(x^2 - 5x + 6) = 0$

$-(x-3)(x-2) = 0$

$x - 3 = 0$ or $x - 2 = 0$

$x = 3$ or $x = 2$

choose $x = 2$ seconds

29. Suppose r(x) = $-x^2 + 26x + 183$ gives the total revenue in dollars made by a bicycle repair shop for repairing x bicycles per day.

a). How many bicycles must be repaired per day to maximize revenue? (2 pts)

$x = \frac{-b}{2a} = \frac{-26}{2(-1)} = \frac{-26}{-2} = 13$

$x = 13$ bicycles

b). What is the maximum revenue per day? (2 pts)

Maximum Revenue $= r(13) = -(13)^2 + 26(13) + 183$

$= -169 + 338 + 183 = 352$

\$352/day

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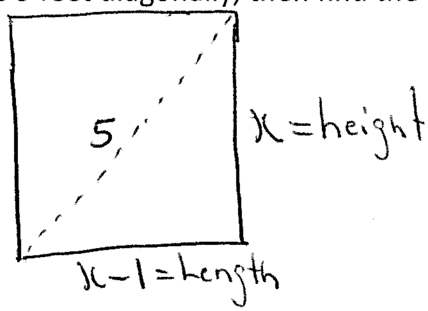
30. The height of a rectangular wall is 1 foot longer than the horizontal distance that it spans. If the wall measures 5 feet diagonally, then find the height and length of the wall. (4 pts)



$$x^2+(x-1)^2=5^2$$
$$x^2+x^2-2x+1=25$$
$$2x^2-2x-24=0$$
$$2(x^2-x-12)=0$$
$$2(x-4)(x+3)=0$$
$$x-4=0 \quad \text{or} \quad x+3=0$$
$$x=4$$

Length of wall is: 3 feet

Height of wall is: 4 feet

31. Truck-A moves 40 mph faster than truck-B. Suppose truck-A travels 350 miles in the same time it takes truck-B to travel 150 miles. Find the speed of each vehicle. (4 pts)

Let truck-B's speed be x and truck-A's speed be $x+40$

	$\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$
Truck-A	$t=\dfrac{350}{x+40}$
Truck-B	$t=\dfrac{150}{x}$

$$\frac{350}{x+40}=\frac{150}{x} \iff$$
$$350x=150(x+40)$$
$$350x=150x+6000$$
$$200x=6000$$
$$x=30 \text{ mph}$$

Truck-A's speed: 70 mph

Truck-B's speed: 30 mph

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32. Liz can paint her house three times as fast as her husband, Eric. If they work together, it takes them 7 hours to paint the house. How long would it take Eric to paint the entire house alone? 4 (pts)

Liz's time $= x$ hrs. ; Eric's time $= 3x$ hrs.

$$\frac{1}{x} + \frac{1}{3x} = \frac{1}{7} \quad ; \quad LCD = 21x$$

$$21x\left(\frac{1}{x} + \frac{1}{3x}\right) = 21x\left(\frac{1}{7}\right)$$

$$21 + 7 = 3x$$

$$28 = 3x$$

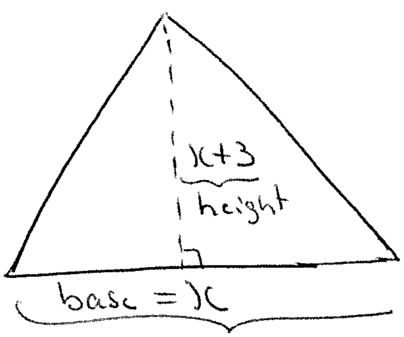
Liz

$$x = \frac{28}{3} \text{ hrs.}$$

Eric's time $= 3\left(\frac{28}{3}\right)$

Eric's time $=$ 28 hours

33. The height of a triangular corn field is 3 miles longer than its base. If the surface area of the field is 14 square miles, then find the base and height of the corn field. (4 pts)



$$\frac{x(x+3)}{2} = 14$$

$$x(x+3) = 28$$

$$x^2 + 3x = 28$$

$$x^2 + 3x - 28 = 0$$

$$(x+7)(x-4) = 0$$

$$x + 7 = 0 \quad \text{or} \quad x - 4 = 0$$

$$x = 4 \quad \text{base}$$

Base of corn field is: 4 miles

Height of corn field is: 7 miles

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Bonus (3pts)

Simplify $(1 + i)^8$ completely to its exact value.

$$(1+i)^8 = \left[(1+i)^2\right]^4 = \left[1+2i+i^2\right]^4 = \left[1+2i-1\right]^4$$

$$= (2i)^4 = 16\cdot i^4 = 16(1) = 16$$

$$(1+i)^8 = 16$$

Have a Wonderful Winter Recess!